

# Coherent Single Spin Source based on topological insulator

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We report on the injection of quantized pure spin current into quantum conductor. In particular, we propose an on demand single spin source generated by periodically varying the gate voltages of two quantum dots that are connected to a two dimensional topological insulator via a tunneling barrier. Due to the nature of the helical states of topological insulator, one or several spin can be pumped out by cycle giving rise to a pure quantized alternative spin current. Depending on the phase difference between two gate voltages, the device can serve as an on demand single spin emitter or single charge emitter. Again due to the helicity of topological insulator, the single spin emitter or charge emitter is dissipationless and insensitive of disorders. The proposed single spin emitter can be an important building block of future spintronic devices.

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Traditional electronics is based on the flow of charge where the spin of the electron is ignored. The emerging technology of spintronics<sup>1</sup> will explore the spin degree of freedom such that the flow of spin, in addition to charge, will be used for electronic applications.<sup>2</sup> Many applications in spintronics have been demonstrated, such as the giant magnetoresistive effect,<sup>3</sup> the spin injection across a magnetic-nonmagnetic interface,<sup>4</sup> and optical manipulation of spin degrees of freedom.<sup>5</sup> Spin degree of freedom can also be used to process quantum information.<sup>6</sup> It is well known that generating qubits or quantum bit that is the unit of quantum information, is one of the basic building blocks for quantum information science. A large variety of candidate qubit systems have been proposed such as photonic qubit<sup>7</sup> and electron qubit.<sup>8</sup> Recently, an on demand coherent single electron source has been produced experimentally<sup>9</sup> and later studied theoretically.<sup>10</sup> By applying ac gate voltage, periodic sequence of single electron emission and absorption on nanoseconds generate a quantized ac current. The single electron transfer between two distant quantum dots has also been demonstrated which paves the way for single electron circuitry.<sup>11</sup> This single electron source can also be used as a qubit in ballistic conductors which is an important step towards 2DEG quantum computer. The major problem in the realization of quantum computers is to identify qubit with long coherence time, spin qubit seems to be the ideal candidate.<sup>12</sup> This is because the spin of electron is weakly coupled to the environment compared with charge degree of freedom, the quantum coherence can be maintained at much longer time scale.<sup>13</sup> It is therefore important to study the transport properties of on demand coherent single spin source that can be used as spin qubit.

Recently, the topological insulator (TI), a new state of matter, has attracted a lot of theoretical and experimental attention.<sup>14–16</sup> The TI has an insulating energy gap in the bulk states which behaves like the general insula-

tor, but it has exotic gapless metallic states on its edges or surfaces. The TI is first predicted in two-dimensional (2D) systems, e.g., the graphene and HgTe/CdTe quantum well. It has been generalized<sup>17</sup> in 3D and confirmed experimentally.<sup>18</sup> The 2D TI has the gapless helical edge states and exhibits the quantum spin Hall effect while in 3D TI the conducting state is helical surface state. This helical edge or surface states are topologically protected and are robust against all time-reversal-invariant impurities. Many interesting physical phenomena have been predicted including Majorana fermion<sup>19</sup>, topological magnetoelectric effect<sup>20</sup>, magneto-optical Kerr and Faraday effects.<sup>21</sup>

In this paper, we report on the injection of quantized pure spin current into quantum conductor. In particular, using the concept of parametric pumping,<sup>22</sup> we study an on demand single spin source generated by periodically varying the gate voltage of two quantum dot (QD) that are connected to a two dimensional topological insulator via a tunneling barrier. Due to the nature of helical state of TI, each QD will generate a fully spin polarized ac current localized near the edge of TI while the direction of the current is controlled by the phase of the gate voltage. As a result of time reversal symmetry, either pure charge current or pure spin current can be generated depending on the phase between two gate voltages of QD. When the phase difference is  $\pi$ , there is a quantized ac spin current with  $n$  spin per cycle pumped out giving rise to a single spin emitter, where  $n$  is an integer. When the phase difference is zero, a spin unpolarized quantized ac charge current is generated with  $n$  charge per cycle. We emphasize that the generated spin current has no accompanying charge current and thus is a pure ac spin source. For this purpose, a quantum transport theory for time-dependent pumped current using non-equilibrium Green's function method in the adiabatic regime is developed. Numerical calculations show that the quantized spin current is robust against the geometric variation of QD as well as the

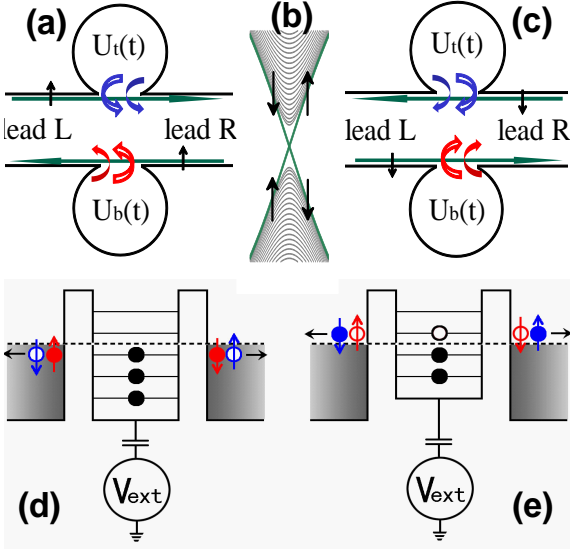


FIG. 1: (Color online) Panel (a) and (c): schematic plot of pumped current in the whole period for the spin up and spin down. Panel (b): band structure of topological insulator. Combining panel (a) and (c), we can get Panel (d) and (e) that depict the generation of pumped pure spin current in the first and second half period, respectively.

weak disorders.

**Model** - In a 2D topological insulator, when the Fermi energy is inside the energy gap, electrons can only be transported through the unidirectional spin locked edge state. In our study, the TI system is coupled with two QPCs at the upper and lower edges whose energy levels are controlled by gate voltages  $U_t(t)$  and  $U_b(t)$ , respectively (see Fig.1). We assume that the amplitudes of two gate voltages are the same but with a phase difference  $\phi$ . As we will discussed below, when  $\phi = 0$ , the system behaves like a coherent single electron source while when  $\phi = \pi$  an alternating quantized spin current is generated, i.e., the system is an on demand single spin emitter without accompanying electric current.

Due to the variation of gate voltage, the current with spin up and down (shown in Fig.1a, Fig.1c, respectively) can be pumped out. In the following discussion, we first focus on the top quantum dot and spin up transport [see Fig.1(a)]. Note that due to the nature of the helical state there is no spin up electron or hole from the top QD to the left lead [see Fig.1(a)]. Assuming that at  $t = 0$ ,  $eV_t$  is minimum, the energy level of QD is below the Fermi level and is completely filled, i.e., filled with two electrons of opposite spin. In the first half period,  $V_t(t)$  decreases from  $V_0$  to  $-V_0$ , the energy levels of QD shifts upward from  $-eV_0$  to  $eV_0$  so that electrons flow out of the QD and into the right lead [see the blue solid arrow in Fig.1(a)]. In the second half period, the energy levels shifts downwards from  $eV_0$  to  $-eV_0$ , consequently, the holes flowing out of the QD to the right lead [see the blue hollow arrow in Fig.1(a)]. It means under certain condition, only one electron (hole) with spin up is pumped out from the

top QD into the right lead giving rise to an alternating quantized spin polarized charge current. Due to the time reversal symmetry, a spin down electron (hole) is pumped out of the QD to the left lead in the first (second) half period [see the blue arrows in Fig.1(c)]. We emphasize that if there were no bottom QD there would be no spin up (down) current in the left (right) lead. Now we consider the independent bottom QD in which the electrons and holes is pumped similar as in the top QD [see the red arrows in Fig.1(a) and (b)]. If the gate voltages of the upper and lower QD are out of phase ( $\phi = \pi$ ), the electron and hole with opposite spin will be pumped into left and right leads in the first [see Fig.1(e)] and second half period [see Fig.1(e)]. As a result, the net charge current in the system is zero. However, there is a pure quantized spin current from left to right lead in the first half period and reverses its sign in the second half period as shown in Fig.1(d) and (e). If the phase different between two gate voltages is zero, there is no spin current and the system generates an alternating quantized charge current.

**theory** - The working principle of coherent single spin emitter (SSE) is based on the 2D topological insulator with a helical spin texture present in momentum space. In order to highlight the functional mechanism of SSE and capture its salient feature, we take the modified Dirac model with a quadratic corrections  $k^2\sigma_z$ , which has the similar properties<sup>23</sup> as HgTe/CdTe. The Hamiltonian is given by  $H(\mathbf{k}) = [H_\uparrow(\mathbf{k}) + H_\downarrow(\mathbf{k})]$ , where

$$H_\uparrow(\mathbf{k}) = H_\downarrow^*(-\mathbf{k}) = A(\mathbf{k}_x\sigma_x - \mathbf{k}_y\sigma_y) + (m + B\mathbf{k} \cdot \mathbf{k})\sigma_z + \epsilon(r)\sigma_0.$$

Here  $\sigma_{x,y,z}$  are Pauli matrices presenting the pseudospin formed by  $s, p$  orbitals and  $\sigma_0$  is a unitary  $2 \times 2$  matrix. The individual spin up Hamiltonian  $H_\uparrow$  and spin down Hamiltonian  $H_\downarrow$  are time reversal symmetric to each other. Since they are decoupled, we can deal with them individually. To carry out numerical calculation, the tight-binding Hamiltonian in square lattice is employed, which is written as<sup>24</sup>

$$H_\uparrow = \sum_{\mathbf{i}} d_{\mathbf{i}}^\dagger [\epsilon_{\mathbf{i}}\sigma_0 + (m - 4t)\sigma_z] d_{\mathbf{i}} + \sum_{\mathbf{i}} \left[ d_{\mathbf{i}}^\dagger (t\sigma_z - i\frac{A}{2a}\sigma_x) d_{\mathbf{i}+\delta_x} + d_{\mathbf{i}}^\dagger (t\sigma_z + i\frac{A}{2a}\sigma_y) d_{\mathbf{i}+\delta_y} \right] + h.c.$$

where  $\epsilon_{\mathbf{i}}$  is a random on-site potential which is uniformly distributed in the region  $[w/2, w/2]$  and  $\mathbf{i} = (\mathbf{i}_x, \mathbf{i}_y)$  is the index of the discrete site in the unit vectors of the square lattice with the lattice constant  $a = 5nm$ .  $d_{\mathbf{i}} = [d_{s,\mathbf{i}}, d_{p,\mathbf{i}}]^T$  with  $T$  denoting transpose,  $d_{s(p),\mathbf{i}}$  and  $d_{s(p),\mathbf{i}}^\dagger$  are the annihilation and creation operators for  $s(p)$  orbital at site  $\mathbf{i}$ . Here  $A/2a = 1.35t$ ,  $m = -0.35t$  and  $t = B/a^2 = 27.5meV$  denote the nearest neighbor coupling strength.

To calculate the time-dependent pumped current, we first examine the adiabatic regime. In the low frequency limit<sup>25</sup>, the system is nearly in equilibrium and the time

dependent pumping parameters are adiabatically added to the Hamiltonian. The charge distribution in the scattering region at any instant is given by

$$Q(\mathbf{i}, t) = -iq \int \frac{dE}{2\pi} [\mathbf{G}^<(E, U(t))]_{ii} \quad (1)$$

where  $U(t)$  is the pumping potential. Since there is no external driving force, the left and right leads have the same Fermi energy. From the fluctuation-dissipation theorem,  $\mathbf{G}^< = (\mathbf{G}^a - \mathbf{G}^r)f(E)$  with  $f(E)$  the equilibrium distribution function for the left and right lead, we can get total charge in the scattering region

$$Q(t) = -iq \int \frac{dE}{2\pi} \text{Tr}[(\mathbf{G}^a - \mathbf{G}^r)f(E)] \quad (2)$$

where  $\mathbf{G}^r(E, \{V(t)\}) = [E\mathbf{I} - \mathbf{H}_0 - \mathbf{\Sigma}^r(E) - \mathbf{U}(t)]^{-1}$ ,  $\mathbf{\Sigma}^r = \mathbf{\Sigma}_L^r + \mathbf{\Sigma}_R^r$  is the self energy from the semi-infinite left and right lead,  $\mathbf{U}$  is diagonal matrix with  $\mathbf{U} = \sum_i \mathbf{V}_i(t)\mathbf{\Delta}_i$ , where  $\mathbf{\Delta}_i$  is the pumping potential profile due to pumping parameter  $V_i(t)$ . Here we assume that the pumping potential profile is constant and  $V_t(t)$  in the upper QD and  $V_b(t)$  in the lower QD have the same amplitude and opposite phase. Due to the variation of pumping potential  $dV_{t/b}(t)$ , the total pumped charge current into all contacts is,

$$dQ(t)/dt = -iq \int \frac{dE}{2\pi} f(E) \sum_i \partial_{V_i} \text{Tr}(\mathbf{G}^a - \mathbf{G}^r) dV_i/dt \quad (3)$$

From Dyson equation  $\mathbf{G}^r = \mathbf{G}^{r,0} + \mathbf{G}^{r,0}\mathbf{U}\mathbf{G}^r$  with  $\mathbf{G}^{r,0} = [E\mathbf{I} - \mathbf{H}_0 - \mathbf{\Sigma}^r]^{-1}$ , we obtain

$$\partial_{V_i} \text{Tr}\mathbf{G}^{r/a} = \text{Tr}[\mathbf{G}^{r/a}\mathbf{\Delta}_i\mathbf{G}^{r/a}] = -\partial_E \text{Tr}[\mathbf{G}^{r/a}\mathbf{\Delta}_i] \quad (4)$$

Note that the bold letter such as Green's functions  $\mathbf{G}^r$ , self energy  $\mathbf{\Sigma}^r$  and potential profile  $\mathbf{\Delta}$  are all matrices which do not commute but can be rotated under the trace operator. Using  $\mathbf{G}^a - \mathbf{G}^r = i\mathbf{G}^r\mathbf{\Gamma}\mathbf{G}^a$  and integrating Eq.(3) by part, we get the total instantaneous charge current from all leads

$$dQ/dt = q \sum_{\alpha} \sum_i \text{Tr} [\mathbf{G}^r \mathbf{\Gamma}_{\alpha} \mathbf{G}^a \mathbf{\Delta}_i] dV_i/dt \quad (5)$$

where  $\mathbf{\Gamma}_{\alpha}$  is the coupling strength between scattering region and the lead  $\alpha$ . Obviously, the above equation gives the current partition into each lead  $\alpha$ . For pumped spin current, we have

$$d\mathbf{S}_{\alpha}/dt = (\hbar/2) \sum_i \text{Tr} [\sigma \mathbf{G}^r \mathbf{\Gamma}_{\alpha} \mathbf{G}^a \mathbf{\Delta}_i] dV_i/dt \quad (6)$$

This equation enables us to calculate the time dependent pumped spin current. We will see below that the SSE emits a single spin in the first half period and reabsorbs a single spin in the second half period.

In order to get a better understanding of the SSE, we integrate the Eq.(7) to get the total spin emitted in half period,

$$S_{\alpha} = (\hbar/2) \int dt \text{Tr} [\mathbf{G}^r(E, V_p) \mathbf{\Gamma}_{\alpha} \mathbf{G}^a(E, V_p) dV_p/dt] \quad (7)$$

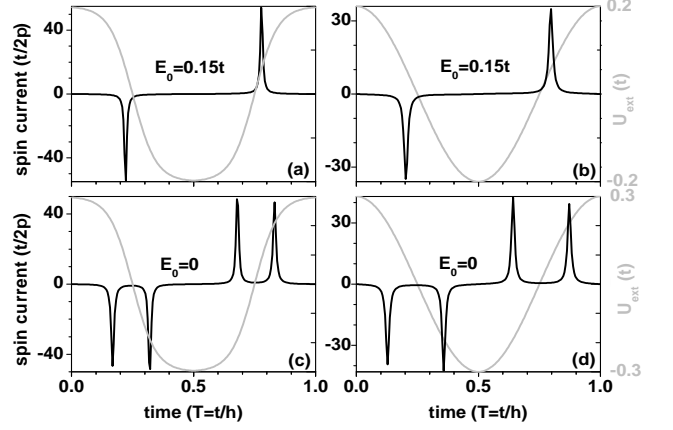


FIG. 2: Pumped current driven by a general ac signal [gray line in panel (a) and (c)] and harmonic signal [gray line in panel (b) and (d)]. Width of lead  $W = 40a$ . The diameter of the circular QD is  $D = 21a$ . The contact width of QPC  $L = 3a$ .

where  $V_p = \sum_i V_i \mathbf{\Delta}_i$ . In one half period, a quantized spin is emitted with the quantized number determined by the energy spectrum of QD and the amplitude of gate voltage. In the whole period, the total spin current is zero.

*Numerical results* - In the adiabatic regime, the energy levels response to the ac gate voltage  $V_{b/t}(t)$  instantaneously. We assume that  $V_{t/b}(t) = V_0 \pm V(t)$  where  $V_0$  is the static gate voltage that can tune energy center of quantum dot  $\epsilon_0$ . In Fig.2, we plot the instantaneous pumped spin current in the whole period for the different energy center  $\epsilon_0$ . It is found the pump current peaks when the Fermi energy sweeps over the energy levels of QD. When  $\epsilon_0 = 0$  the Fermi energy is just in the middle of highest occupied level and lowest unoccupied level in equilibrium state, i.e., at time  $\tau = 0.5T$ . When  $\tau$  change from 0 to  $T$ , the Fermi level scans over the first lowest electron level and the first highest hole levels, so two peaks appear in Fig.2c and Fig.2d. Note that due to the coupling of the leads, the energy levels of QPC are different from the isolated quantum dot. For example, the first level of isolated QD  $\epsilon_{\pm 1}^{iso} = \pm 0.19t$  and the first renormalized energy level of opening QD  $\epsilon_{\pm 1} = \pm 0.15t$ . Through the static external gate voltage, we can tune the level of the quantum dot. When  $\epsilon_0 = 0.15t$ , Fermi energy is in line with the first hole level in equilibrium moment, when amplitude of ac signals  $V_{b/t}$  is small, in the whole period, the Fermi energy can only scan over the first hole level, then one peak appears in Fig.2a and Fig.2b. For panel (a), (b), (c) and (d), the quantum value of quantized spin in a half period is 1.015, 1.015, 1.957, 1.957, respectively. The different ac signals (such as the harmonic and non-harmonic ac signal and harmonic signal as in Fig.2 and any other different ac signals) induces the different instantaneous currents, however they all drive the same single spin in a whole period. So this model can be used

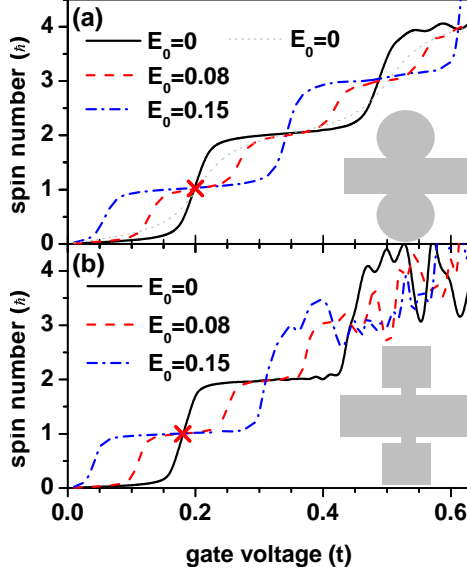


FIG. 3: (Color online) The number of spin pumped out of QD in the first half period vs magnitude of the ac gate voltage. Different curves correspond to different energy levels  $E_0$  of the topological insulator system. The diameter of the circular QD is  $D = 21a$  and contact width  $L = 3a$  (thick lines) and  $L = 7a$  (thin gray dotted line). The size of square QD is  $20a * 20a$  and contact width  $L = 7a$ .

as a general spin source in spintronic devices.

In the following we will show the single spin source can be realized in all QPC with different geometric shapes of QD. In Fig.3, we plot the quantized spin accumulated in a half period. We consider two QD: a circular QD [panel (a)] and the square shaped QD [panel (b)]. For different  $\epsilon_0$ , there are three representative configurations of quantization: when Fermi energy of the lead  $E_F = 0$  lies between  $\epsilon_{+1}$  and  $\epsilon_{+1}$ , i.e.,  $\epsilon_0 = 0$ ,  $V_{b,t}$  scan an even number of energy levels in a half period and the quantized spin with even number of spin are generated (see black solid lines in Fig.3). When  $E_F$  is roughly in line with renormalized energy level of QD, i.e.,  $\epsilon_0 = 0.15t$ , the number of spins are 1,3,5... (see dash dotted lines in Fig.3), otherwise, the number of spins are integers 1,2,3.... Furthermore, from Fig.3a, we can see the weaker the coupling between TI and quantum dot, the longer the quantized spin plateaus is since the band width is small for the weak

coupling [comparing the black solid line and gray dotted line in Fig.3(a)]. Besides, in the weak coupling case the quantum plateaus are not so flat because the electron is difficult to escape from QD.

In Fig.4, we plot the spin accumulation in and before the first quantized plateau (the red cross in Fig.3) vs disorder strengths for different  $\epsilon_0$ . It is shown that although fluctuation of quantized spin roughly linearly increases with increasing disorder, the averaged values of quantized spin are hardly changed by disorder, especially for the quantized values in the quantum plateaus. It means

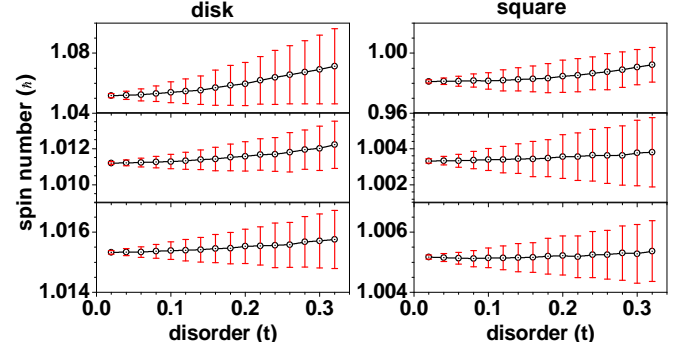


FIG. 4: The number of spin at the gate voltage marked red cross in Fig.3 vs disorder strengths for different energy levels  $E_0$ . The left and right columns are corresponding to the circular and square shaped QD.

this spin emitter is robust against the disorder scattering.

In conclusion, we have proposed a single spin emitter that is driven by two ac gate voltages. Due to the helical feature of topological insulators, an alternative pure spin current with quantized spin per cycle can be generated. Importantly by tuning the phase difference between two gate voltages, either ac quantized spin current or ac quantized charge current can be pumped out. Our numerical results show that this quantized single spin emitter is robust again disorders and variation of device shapes.

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